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Valuation of a Company using Time Series Analysis

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Abstract: In this paper we present an approach to value-based management of companies using time series analysis. We present a technique for projecting cash flows in order to calculate the company value using time series analysis. We consider a new, indirect approach and a direct approach of projecting cash flows. We analyse both models from the perspective of value-based management. Finally, company value is calculated for both models, as a point estimate and as a distribution function respectively. As shown in the article, the distribution function of corporate value is a normal distribution function. On this basis, it is possible to apply all instruments of value-at-risk analysis.

Keywords: time series analysis, value-at-risk analysis, value-based management
JEL Classification: C53, G32

1 Introduction

1.1 Motivation

In addition to competing for customers, companies must compete for investors who are willing and able to invest in the company's business model. Investors expect profitable products and therefore an attractive income-to-equity-ratio. This forces the management of the company to manage the different sources of surplus successfully and sustainably. Consequently, the management requires some method of measuring the success of an instance of capital expenditure from the shareholder's perspective.

Value-based management is one such approach, meeting all of the above requirements. Value-based management decides whether a potential course of action increases the value of invested capital, namely by achieving a rate of return that exceeds the cost of capital. The capital required and the risk of

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capital expenditure are integrated into this assessment. When value-based management is applied in a company, all management decisions are made with the single objective of maximising company value, which is calculated by summing all future cash flows. Therefore, projecting cash flows is a critical aspect of value-based management.

1.2 Objective and Main Result of the Paper

The objective of this paper is to present a technique of projecting cash flows that may be used as a basis for calculating company value using time series analysis. We consider a new, indirect approach to projecting cash flows, which models the cash flow-defining business parameters such as sales, material-related expenditure, or personnel-related expenditure as autoregressive moving average processes, from which the corresponding cash flows may be derived. This yields detailed a financial plan for the business model being considered, allowing the company to be successfully managed and controlled. We will also study a direct approach to forecasting cash flows. We analyse both models from the perspective of value-based management. Finally, we will calculate company value using each model in turn.

The main result of this paper is a derivation of company value as both a point estimate and a distribution function, using time series modelling. This is a fundamentally new approach to evaluating companies. This new result is achieved in both the new, indirect and the direct approaches to projecting cash flows. By considering the distribution function of the corporate value, which we prove is a normal distribution function, it will then become possible to apply the instruments of value-at-risk analysis, enabling a deeper understanding of the business, and enabling successful value-based management.

The paper is organised as follows: in section two, we give an overview of current literature. In section three, the theoretical model is explained. In section four, the practical data and the business model of a company are recorded. In section five, we project cash flows by means of time series analysis. Both the indirect and the direct approach are considered. We study the specification of the models and also perform extensive diagnostics on both models. We then project cash flows using both models. In section six, the company value is derived. We obtain both a point estimate and a distribution function. In section seven, the extrapolation problem is discussed elaborately. In section eight, the model is expanded to cover also a company of another industry. Our findings are summarised in section nine. Finally, further avenues of research are suggested in section ten.

2 Review of Literature

2.1 Literature for Projecting a Single Business Parameter

The current literature may be categorised into two sections. The first section discusses models for projecting a single cash flow-defining business parameter such as sales or cost using time series analysis. No further derivation of cash flow or cash flow analysis is performed. Hattingh and Uys (2014) present a model for projecting sales using time series analysis. This projection is used to derive the optimal product portfolio. Zadeh, Sepehri, and Farvaresh (2014) provide a sales projection using time series analysis as a means for calculating the optimal inventory level, which is important when balancing the capital cost of excessive inventory against consumer dissatisfaction due to product shortage. To give an accurate forecast of company sales, it is necessary to analyse market trends correctly. Bagheri, Mohammadi, and Akbari (2014) present a model that predicts future market trends by applying time series analysis. They derive the equilibrium price in different economic scenarios. Sales for a given company depend on the stage of business cycle that this company is currently in. Chen (2007) demonstrates a technique for measuring business cycle turning points using non-stationary time series analysis. In this way, the structure of business cycles can be studied. Sales also depend on economic parameters such as exports. Farooqi (2014) forecasts exports for a given country by fitting a time series model. In addition to sales, cost is also a defining parameter for the cash flow. Vogt et al. (2014) develop a model for projecting service cost by applying time series analysis and multiple regression. Yip, Fan, and Chiang (2014) study a model for projecting maintenance costs by comparing a time series and a general regression approach. They test the forecasting accuracy of both approaches. Hwang (2010) presents a model for forecasting the productivity and therefore the production cost of a company. He compares a time series model with other, different forecasting models and demonstrates that the time series approach is superior to the other models. Golyandina and Korobeynikov (2014) describe how to implement spectrum analysis of time series analysis to forecast cash flow-defining business parameters using the software R.

2.2 Literature for Projecting the Cash Flow

The second section of the literature describes models for projecting the cash flow directly. In these discussions, no projections of cash flow-defining business parameters are made. The models are fitted to the cash flow itself. Therefore, no financial plan is produced for the business model being studied. McIntosh (1990) gives an overview about some basic techniques for projecting the cash flow directly using time series analysis. Kim and Kross (2005) analyse a model for predicting annual cash flows from actual earnings. Lorek and Willinger (2008) develop a model for projecting quarterly cash flows based on time series analysis. Lorek and Willinger (2009) demonstrate that models based on time series analysis and therefore based on past cash flows are superior to other types of model. Lorek and Willinger (2011) discuss different time series and regression models for a quarterly projection of cash flows. They compare the forecasting accuracy of these models by calculating the mean absolute percentage error. They show that the model of Brown-Rozeff is the best model for predicting quarterly cash flows.

In contrast to the existing literature, we will model future cash flows by projecting the cash flow-defining business parameters such as sales, material-related expenditure, and personnel-related expenditure using time series analysis, and then derive the cash flows themselves by a new, indirect approach. In this way, we obtain a comprehensive financial plan for the business model in question. We will compare this new, indirect approach with a direct approach to projecting cash flows. As an important extension to the existing literature, we will record a derivation of the company value from these future cash flows. Since the value is calculated both from an optimal projection, and from a stochastic process of time series, we will obtain both point estimates and distribution functions as expressions for the company value.

3 Model

The objective of value-based management is to maximise a company's value. To achieve this, every management decision must aim to increase this value. This is the ultimate decision criterion that governs whether potential investments are realised, and if so, in which order of priority. To calculate a company's value for purposes of value-based management, the following formula is used:[1]

1 The formula represents the equity approach of the discounted cash flow method. It would also be possible to apply the entity approach by considering the cash flows to entity and subtracting the debt. Both approaches result in the same value for any given company.

$$CV = \sum_{t=1}^{T} \frac{CF_t}{(1+k)^t} + \frac{TV_T}{(1+k)^T}, \quad [1]$$

where CV, company's value; T, time horizon; CF_t, cash flow to equity at time t; k, cost of capital; TV_T, terminal value at time T.

In order to apply this formula, being able to project future cash flows is especially critical. The company value depends primarily on these cash flows. Therefore we consider two approaches – one indirect and one direct – to deriving these cash flows.

3.1 Model 1 as Indirect Approach

In the indirect[2] approach, we model all cash flow-defining business parameters such as sales, material-related expenditure and personnel-related expenditure as autoregressive integrated moving average processes ARIMA(p, d, q). An ARIMA(p, d, q) is a stochastic process X_t satisfying the following fitted model:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \varepsilon_t - \beta_1 \varepsilon_{t-1} - \ldots - \beta_q \varepsilon_{t-q}, \quad [2]$$

where X_t, endogenous variable of interest such as cash flow or sales; α_i, coefficient associated with the "lagged" value X_{t-i}; β_i, coefficient associated with the "lagged" residual term ε_{t-i}; ε_t, residual term for time t.

Autoregressive moving average processes differ substantially from ordinary least square methods: the actual value of an observed variable is dependent on the past values of this variable and the related error terms instead of causal predictor variables. We consider a general setting in which we include past values until p previous periods and error terms until q previous periods. We call p and q the autoregressive and moving average time lag respectively. For given data p and q and therefore the corresponding model can be estimated by calculating the Akaike's Information Criterion (AIC), which is essentially equal to the negative log-likelihood function plus a correctional term representing the model order. The model with the smallest AIC is then chosen. Furthermore the model can be estimated by deriving the Bayesian Information Criterion (BIC), which is basically equal to the negative log-likelihood function plus another

2 We consider an indirect approach because (Hülss et al. 2012) have shown that for random walks and Wiener processes, models that project cash flow indirectly are superior to models that project cash flow directly. We expand their analysis by using a time series analysis approach instead of a Markov model.

correctional term representing the model order. Also the model with the smallest BIC is then chosen. We provide both measures to execute model selection on a broad basis.

Autoregressive moving average techniques assume that the structure of development concerning the observed variable is the basis for its future projection. There exist different studies[3] demonstrating that a cash flow projection based on autoregressive moving average technique has a high forecasting accuracy.

There is an assumption underlying ARIMA time series forecasting methods that there is a characteristic steady state where the mean and variance of the statistic of interest is constant. This steady state is referred to as the stationary state. It is recognized that most real world time series are anything but stationary. The statistic of interest over time exhibits upward or downward trends, seasonal patterns and other systematic movements that are thought to underlie the observed deviation from stationarity. So the ARIMA process becomes one of determining what factors cause a series to deviate from stationarity. ARIMA in effect works backward. We model the effect of trend, seasonality and systemic error by starting with a clearly non-stationary series (one that contains trends, seasonal effects and other systemic error) and seeing what sorts of transformations will convert it into a stationary one. The transformations needed to remove trend, seasonality and other systemic causes are then used to implicitly model the non-stationary "real world" series in making forecasts.

If the original data are not stationary due to a linear or polynomial trend it is necessary to differentiate the original data resulting in a process of differences. When doing this d times we eliminate the trend and finally end up with stationary observations.[4] To compute d we apply a unit root test, testing if the characteristic equation of the process has a unit root. If so, it would be necessary to further differentiate the process. We can apply eq. [2] either to the original data ($d = 0$), or to the differences of order d ($d \geq 1$).

An ARIMA(p, d, q) model is fitted to every cash flow-defining business parameter. Then, the cash flow is derived from these business parameters. With this setup, we obtain a complete financial plan for the business model in question. All income and expense items are integrated into the plan in full detail. This is important for managing and controlling a business department or a company. Every income and expense source can be analysed and projected to ensure that the business model is successfully carried out. We denote this indirect approach as Model 1.

3 See for example Lorek and Willinger (2009), Lorek and Willinger (2011) or (Hülss et al. 2012).
4 If there is a polynomial trend of order d we have to differentiate d times to get stationary observations.

3.2 Model 2 as Direct Approach

In the direct approach, we consider the model of Brown-Rozeff. The model was originally developed by Brown and Rozeff (1979) as a candidate prediction model for quarterly earnings per share. Their model has the following form:

$$CF_t = CF_{t-4} + \varphi(CF_{t-1} - CF_{t-5}) + \varepsilon_t - \theta\varepsilon_{t-4}. \qquad [3]$$

This model is a combination of an autoregressive model and a seasonal moving average model with first order differences. To understand this, we consider the concept of lagged operators. We define the autoregressive operator $(1-\Phi)$, the seasonal autoregressive operator $(1-\Phi_4)$, the moving average operator $(1-\Theta)$ and the seasonal moving average operator $(1-\Theta_4)$ as follows:

$$\begin{aligned}(1-\Phi)CF_t &= CF_t - \alpha_1 CF_{t-1},\\ (1-\Phi_4)CF_t &= CF_t - \alpha_4 CF_{t-4},\\ (1-\Theta)\varepsilon_t &= \varepsilon_t - \beta_1\varepsilon_{t-1} \text{ and}\\ (1-\Theta_4)\varepsilon_t &= \varepsilon_t - \beta_4\varepsilon_{t-4}.\end{aligned} \qquad [4]$$

Applying the seasonal autoregressive or moving average operator we divide a season in four periods, i. e. we divide a year in four quarters. It is important to note that the previous time of time t in a seasonal model is t–4.

We illustrate this concept for two simple examples: an ARMA(1, 1) model is given by applying once the autoregressive operator and once the moving average operator. Therefore we have

$$\begin{aligned}(1-\Phi)X_t &= (1-\Theta)\varepsilon_t \text{ and accordingly}\\ X_t - \alpha_1 X_{t-1} &= \varepsilon_t - \beta_1\varepsilon_{t-1} \text{ and}\\ X_t &= \alpha_1 X_{t-1} + \varepsilon_t - \beta_1\varepsilon_{t-1},\end{aligned} \qquad [5]$$

which is exact the form [2] for an ARMA(1, 1) model.

A seasonal ARMA(1, 1) model is given by applying once the seasonal autoregressive operator and once the seasonal moving average operator. Therefore we have

$$\begin{aligned}(1-\Phi_4)X_t &= (1-\Theta_4)\varepsilon_t \text{ and accordingly}\\ X_t - \alpha_4 X_{t-4} &= \varepsilon_t - \beta_4\varepsilon_{t-4} \text{ and}\\ X_t &= \alpha_4 X_{t-4} + \varepsilon_t - \beta_4\varepsilon_{t-4},\end{aligned} \qquad [6]$$

which is exact the form [2] for a seasonal ARMA(1, 1) model.

For the model of Brown-Rozeff we consider the seasonal differences $(CF_t - CF_{t-4})$ and apply once the autoregressive operator and once the seasonal moving average operator resulting in

$$(1-\Phi)(CF_t - CF_{t-4}) = (1-\Theta_4)\varepsilon_t \text{ and accordingly}$$
$$(CF_t - CF_{t-4}) - \varphi(CF_{t-1} - CF_{t-5}) = \varepsilon_t - \theta\varepsilon_{t-4}. \quad [7]$$

From this eq. [3] follows directly. Applying the Brown-Rozeff model the actual seasonal cash flow difference depends on the seasonal cash flow difference before one period and the error made when estimating the seasonal difference before one season.

There exists a short notation called the Box-Jenkins notation where a combination of a normal ARIMA model and a seasonal ARIMA model can be written as $(p, d, q) \times (P, D, Q)$. The part (p, d, q) refers to the normal and the part (P, D, Q) refers to the seasonal ARIMA model with corresponding order. Using the Box-Jenkins notation the Brown-Rozeff model can be written as $(1, 0, 0) \times (0, 1, 1)$ where the first term is a normal ARIMA(1, 0, 0) model and the second term is a seasonal ARIMA(0, 1, 1) model.

Lorek and Willinger (2008) provide evidence that this model is adequate for quarterly cash flow projection.[5] Lorek and Willinger (2011) compare the forecasting accuracy of this model for purposes of quarterly cash flow prediction with that of a multivariate regression model by calculating the mean absolute percentage error. They demonstrate that the Brown-Rozeff model is the best model for predicting quarterly cash flows in the short-term and in the long-term, based on a large sample of companies. The model is parsimonious in nature because it relies upon lagged values of the quarterly cash flows to derive its model structure. We denote this direct approach as Model 2.

4 Data

We consider a company of the software industry. This company develops and produces software for communication systems. The business model is based on a classical project-based business. Since many milestones are in the last quarter of each year, the corresponding sales also occur in the last quarter of each year. As a consequence, the cash flows in the last quarter of each year are considerably higher than the cash flows in the first to third quarters of each year. This is a typical example of a seasonal business, and for that reason we need to use a model based on quarterly cash flows.

5 Due to the nature of the business model, the data we analyse is organised by quarter (see Section 4). Therefore we will fit a model for quarterly cash flows.

The data we will analyse results from the period 2010–2014. This period corresponds to the time that the current business model has been in operation. Before 2010, the business model was different, so we cannot compare figures resulting from pre-2010 to more recent ones. Since we fit the models on a quarterly basis, the data is from the first quarter of 2010 to the fourth quarter of 2014. Consequently we have $t = 20$ past data-points for fitting our models.[6] For each quarter we have sales, material-related expenditure and personnel-related expenditure. Sales result from different communication software projects. Material-related expenditure includes investments in hardware and software as well as payments for external services. Personnel-related expenditure refers to the salaries of the IT specialists developing and programming the complex software solutions. Other income and expenditure are negligibly small, so that the cash flow is mainly governed by these business parameters. We fit Model 1 to these cash flow-defining business parameters and project the cash flow indirectly on a quarterly basis. We will also fit Model 2 to the quarterly cash flows and project the quarterly cash flows directly.

5 Projection of Cash Flows

5.1 Specification of the Model

We now fit Model 1 to the data. This means that we fit an ARIMA(p, d, q) model to the sales, material-related expenditure and personnel-related expenditure in turn. The fitting is performed on a quarterly basis. Since other sources of income and expenditure are negligibly small, we can derive the cash flow indirectly from these cash flow-defining business parameters.[7] We demonstrate the process of fitting the model for sales. The same process is then applied to fit the models for material and personnel-related expenditures. For these parameters, we will only summarise the results.

We consider the sales of the company from the first quarter of 2010 to the fourth quarter of 2014. To find the right order of differences d we apply a unit

6 If we would use a model based on yearly cash flows we only had $t = 5$ past data-points for fitting our models, which are not enough to fit the models adequately. The model fitting and the forecasting accuracy are not satisfactory.

7 In the case that there are further income or expenses, additional ARIMA models have to be fitted to these data. The main results remain valid.

root test: the augmented Dickey-Fuller test. This test evaluates the hypothesis that there exists a unit root. If so, it would be necessary to further differentiate the process. After deriving the first order differences, the test rejects the hypothesis of a unit root for a significance level of $\alpha = 0.05$. So, no further differences need to be considered: we have $d = 1$. One intuitive way to interpret this result is that the first order differences of the lagged values provide all the useful information needed for accurate forecasting.

For specification of the model we have a graphical and a probability theory based analytical method. Applying the graphical method we compare the empirical and the theoretical autocorrelation function and partial autocorrelation function in order to select the autocorrelation part of the model, which is the p term, and the moving average part of the model, which is the q term. Thereby the autocorrelation function is used to estimate the q term and the partial autocorrelation function is used to estimate the p term. Applying the probability theory based analytical method we calculate AIC and BIC in order to select the best fitting model. We analyse the autocorrelation function and the partial autocorrelation function of differentiated sales, both of which are shown in Figure 1.

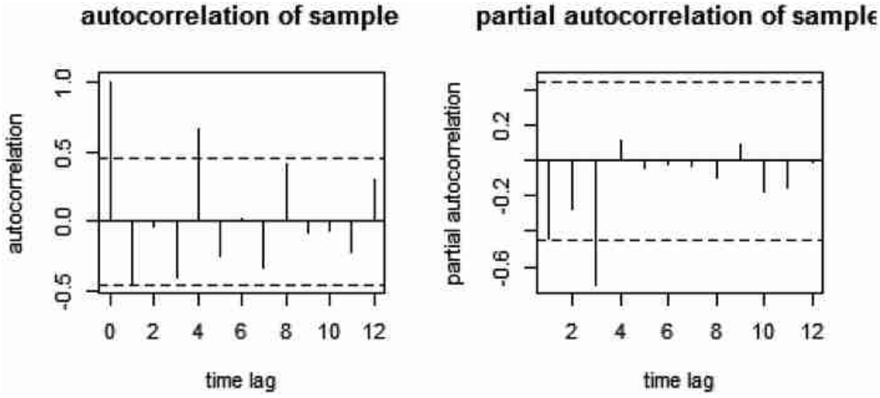

Figure 1: Autocorrelation function and partial autocorrelation function of differentiated sales.

It can be seen that the autocorrelation function has the shape of a sinusoidal wave to infinity, whereas the partial autocorrelation has significant values until lag three and non-significant values for larger lags. Therefore we may conclude that $p = 3$ and $q = 0$ is the order of the model. More formally, we consider the AIC. The model with the smallest AIC is then chosen. In our example, the ARIMA (3,1,0) has an AIC = 360.15, which is the smallest for all suitable p and q.

Furthermore we consider BIC and select the model with the smallest BIC. The ARIMA(3, 1, 0) has a BIC = 360.29, which is the smallest for all suitable p and q. Therefore model selection based on AIC and BIC is consistent. Now the model parameters can be estimated by applying the maximum likelihood method. The results are shown in Table 1. In a similar fashion, we specify ARIMA models for material and personnel-related expenditures on a quarterly basis and estimate the model parameters using the maximum likelihood method. The results are shown in Table 1.

Table 1: Specification and diagnostics of Model 1 and Model 2.

Model 1	**Structure**	α_1	α_2	α_3	β_1	**Kolmog. Smirnov**	**Breusch Pagan**	**Ljung Box**
Sales	ARIMA(3,1,0)	−0.9385	−0.9143	−0.8788		×	×	×
Material	ARIMA(0,1,1)				−0.7653	×	×	×
Personnel	ARIMA(3,1,0)	−0.9059	−0.7148	−0.7537		×	×	×
Model 2	**Structure**			φ	θ	**Kolmog. Smirnov**	**Breusch Pagan**	**Ljung Box**
Cash flow	seasonal ARIMA(1,0,0) × (0,1,1)			−0.2272	0.1846	×	×	×

Now we fit Model 2. For this model, no further specification is required than that given by eq. [3]. We can immediately estimate the model parameters by applying the maximum likelihood method. The results are also shown in Table 1.

5.2 Diagnostics of the Model

To ensure that the models fit well to the data we now provide a detailed residual analysis. We analyse the normal distribution, the homoscedasticity and the independence of the residuals. First, we argue graphically, then, we substantiate the analysis with statistical tests. We begin with Model 1 and study the differentiated sales. Consider the normal distribution plot in Figure 2, where the quantiles of the sample and the theoretical normal distribution quantiles are plotted together. Since the points lie approximately on a line, the residuals are normal distributed. We have a mean of zero because the line passes through the origin. The variance is larger than the variance of the standard normal distribution because the slope of the line is greater than one. Now, we apply the Kolmogorov-Smirnov test. This test compares the empirical distribution function of the residuals to the distribution function of

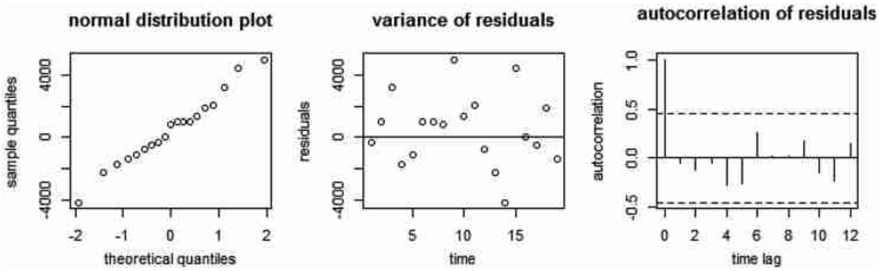

Figure 2: Residual analysis of differentiated sales.

a normal distribution using an appropriate distance measure. This test does not reject the hypothesis of a normal distribution for a significance level of $\alpha = 0.05$.

The graph depicting the variance of residuals in Figure 2 plots the residuals as a function of time. It can be seen that the variance of the residuals remains stable over time, i. e. the residuals are homoscedastic. To test for homoscedasticity, we apply the Breusch-Pagan test. This test models the squared residuals with a linear regression model as a function of time. If there is a significant time-dependency, the test rejects the hypothesis of homoscedasticity. Again, this test does not reject the hypothesis of homoscedasticity for a significance level of $\alpha = 0.05$.

By analysing the autocorrelation of residuals plot in Figure 2, it can be seen that the autocorrelations are not significant for any value of time lag. Therefore, we can conclude that the residuals are independent. As a formal test of independence, we consider the portmanteau test of Ljung-Box. This test sums all empirical autocorrelation coefficients up to a certain time lag. If this test statistic is too large, the hypothesis of independence is rejected. Again, the test does not reject the hypothesis of independence at all suitable time lags for a significance level of $\alpha = 0.05$. For material and personnel-related expenditure, the graphs look similar and all tests for normal distribution, homoscedasticity and independence do not reject the corresponding hypotheses for a significance level of $\alpha = 0.05$. The results are shown in Table 1.

We now analyse Model 2. In Figure 3, the graphs discussed above are presented. It can be seen graphically that the required normal distribution, homoscedasticity and independence of the residuals are fulfilled. Also, the statistical tests for normal distribution, homoscedasticity and independence do not reject the corresponding hypotheses for a significance level of $\alpha = 0.05$. The results are shown in Table 1.

Finally, we present all results for Model 1 and Model 2 in Table 1.

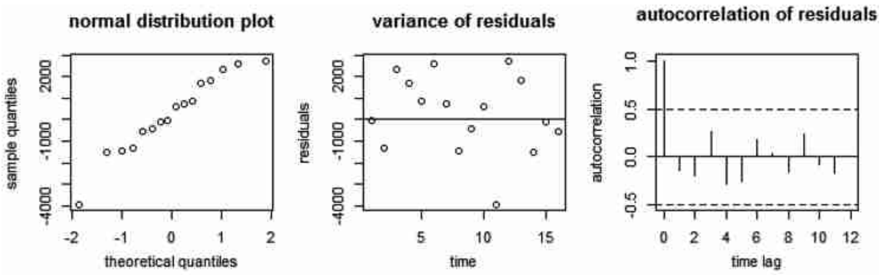

Figure 3: Residual analysis of cash flow in Model 2.

5.3 Comparison of the Models

5.3.1 Model Fitting

We now compare Model 1, which derives cash flow indirectly, and Model 2, which derives cash flow directly, in terms of model fitting and forecasting accuracy. By considering Figure 4, we see that Model 1 is a good fit for the cash flow-defining business parameters: sales, material-related expenditure and personnel-related expenditure. Only the small peaks of material-related

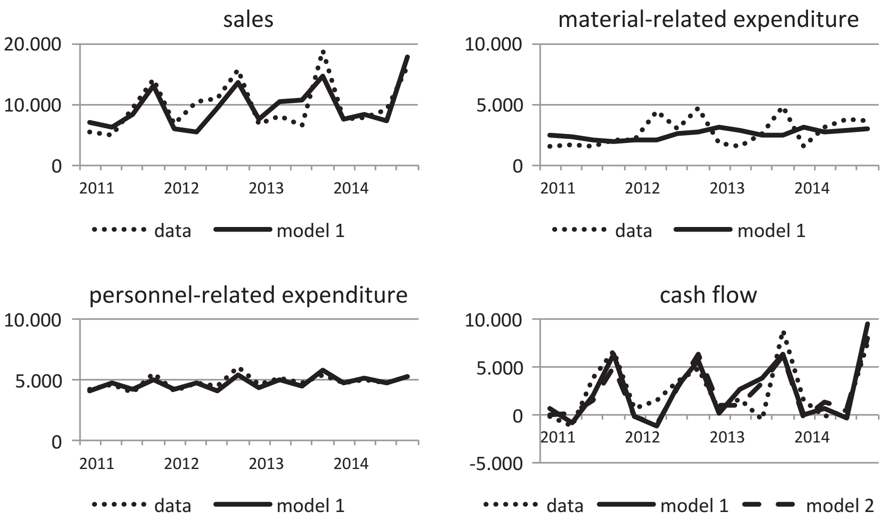

Figure 4: Model fitting of Model 1 and Model 2.

expenditure are not adequately modelled due to the moving average structure. Also, the derived cash flow shown in the last diagram is a good fit for the data. Model 2 also provides a good fit for the data, as can be seen in the last diagram. It is interesting to note that both indirect and direct approaches of modelling the cash flow result in essentially the same fitted values for this cash flow, as can be seen in the last graph. As a formal fitting measure, we calculate the mean absolute percentage error (MAPE):

$$\mathrm{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right|, \qquad [8]$$

where n, number of observations; A_i, actual value; F_i, fitted value.

We can calculate the MAPE on a yearly and on a quarterly basis. The MAPE on a yearly basis is calculated by comparing the yearly actual and fitted values, the MAPE on a quarterly basis is derived by considering the quarterly actual and fitted values. When evaluating a company the yearly future cash flows are important. The quarterly distribution of these yearly cash flows is irrelevant. We realise this by considering eq. [1]. Therefore we benchmark our models by calculating the MAPEs on a yearly basis. We also provide the MAPEs on a quarterly basis to show that all results stated for MAPEs on a yearly basis still remain valid for MAPEs on a quarterly basis. Since quarterly excesses and deficits may partly balance each other, yearly MAPEs are generally smaller than quarterly MAPEs.[8]

For Model 1, we find the following yearly MAPEs: sales 8 %, expenditure for material 18 %, expenditure for personnel 2% and derived cash flow 18 %. For Model 2, we find a yearly MAPE of 17 % for the cash flow. So, in terms of

8 Another effect is the following: when calculating the yearly MAPE and therefore comparing yearly actual and fitted values, we may consider the percentage difference between a yearly actual and a fitted value as a weighted sum of quarterly percentage differences between actual and fitted values with weights equal to quarterly actual values. This means that if a quarter has a relevant impact to the company value it gets a higher weight than a quarter with no impact. Therefore the yearly MAPE tends to be small if the model is suitable for deriving the company value. When calculating the quarterly MAPE and therefore comparing quarterly actual and fitted values, we sum up arithmetically all quarterly percentage differences between actual and fitted values with same weights. This means that although a quarter has no relevant impact to the company value it gets the same weight as a quarter with high impact. Therefore quarterly MAPEs are not a suitable benchmark for models in order to evaluate a company. For that reason we mainly focus on yearly MAPEs when evaluating a company. Our models fit very well to the data especially in the main quarter, which is the fourth quarter. Therefore quarterly percentage differences between actual and fitted values are small for observations with a high weight. Thus the yearly MAPE tends to be small because the models are suitable for deriving the company value.

cash flow fitting ability, both models are equally suitable. The cash flow, which is a critical variable for value-based management, is well-fitted in both models.[9]

5.3.2 Forecasting Accuracy

We now analyse the forecasting accuracy of both models. To do so, we fit Model 1 and Model 2 to the restricted past time period extending from the first quarter of 2010 to the fourth quarter of 2013. We then use these models to forecast the period extending from the first quarter of 2014 to the fourth quarter of 2014. For Model 1, we do this for all cash flow-defining parameters and then derive the cash flow indirectly. For model 2, we forecast the cash flow directly. The results are shown in Figure 5. It can be seen that Model 1 has a good forecasting accuracy for all cash flow-defining parameters and for the derived cash flow, as does Model 2 with the direct cash flow approach.

As formal forecasting accuracy measure, we consider again the MAPE, but now we denote F_i the forecasted value in eq. [8]. For Model 1, we find the following yearly MAPEs: sales 1 %, material-related expenditure 4 %, personnel-related expenditure 0 % and derived cash flow 5 %. For Model 2 we have a yearly MAPE of 1 % for the cash flow. So in terms of cash flow forecasting accuracy, Model 2 performs slightly better than Model 1. The cash flow forecasting accuracy, which represents an important performance measure in value-based management, is essentially satisfactory in both models.[10]

9 We have the following quarterly MAPEs for Model 1: sales 19 %, expenditure for material 39 %, expenditure for personnel 5 % and derived cash flow 62 %. For Model 2, we find a MAPE of 59 % for the cash flow. Again, in terms of cash flow fitting ability, both models are equally suitable. The cash flow is well-fitted in both models.

10 We find the following quarterly MAPEs for Model 1: sales 13 %, expenditure for material 39 %, expenditure for personnel 1 % and derived cash flow 83 %. For Model 2, we find a MAPE of 61 % for the cash flow. Again, in terms of cash flow forecasting accuracy, Model 2 performs slightly better than Model 1. The cash flow forecasting accuracy is essentially satisfactory in both models. It is important to note, that the quarterly MAPE for the cash flow of Model 1 is comparable to the MAPE of the multivariate regression model analysed by Lorek and Willinger and the quarterly MAPE for the cash flow of Model 2 is comparable to the Brown-Rozeff model considered by Lorek and Willinger, see Lorek and Willinger (2011, 80). So our Model 1 and Model 2 have the same forecasting accuracy as the best models for predicting quarterly cash flows in the literature.

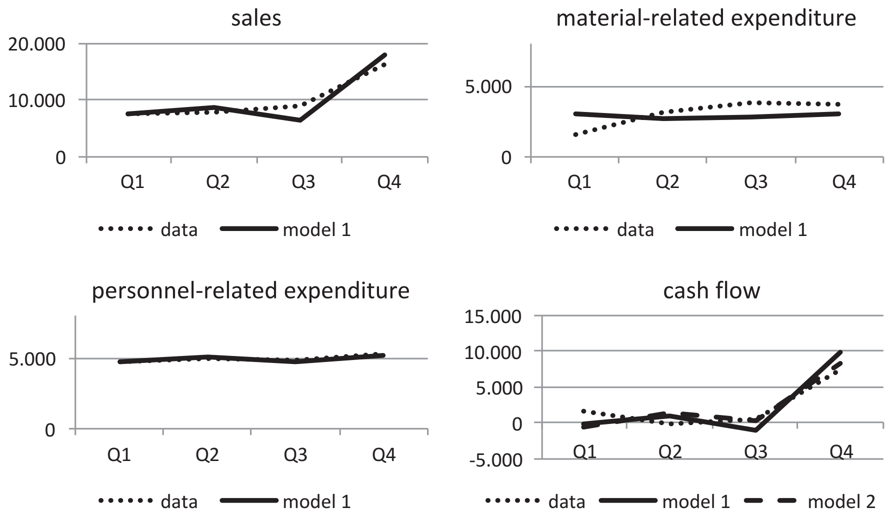

Figure 5: Forecasting accuracy of Model 1 and Model 2.

Considering both models from the perspective of value-based management, the conclusion is as follows: using Model 1 the cash flow is sub-divided into its operative parts, the cash flow-defining business parameters. Therefore it is possible to manage operatively and control single business units such as the marketing department with the corresponding sales projections, the production department with the corresponding projections of material-related expenditure, and the human resources department with the corresponding projections of personnel-related expenditure. A detailed financial plan integrating all important and operative key performance indicators is produced. The goodness of fit of this model and the forecasting accuracy at this operative level are good. So Model 1 should be used for operative purposes in the context of value-based management.

When applying Model 2, the cash flow itself is modelled directly according to its own structure. The forecasting accuracy of the direct approach is slightly better than that of the indirect approach, in which single mistakes arising from fitting each model to its respective cash flow-defining parameter risk adding up to a larger mistake in the cash flow. Since the forecasting accuracy is of high importance when calculating the company value, Model 2 is suitable when the focus is on the financial ratios of a strategy in relation to company value. So Model 2 should be used for strategic purposes in the context of value-based management.

5.4 Projection Using the Model

Now that we have shown that both models are adequate for projecting the business model from both an operative and a strategic perspective, we will apply both models to provide the cash flow projection required for calculating company value. We distinguish between a projection made based on the optimal projection, and a projection made based on a stochastic process. This difference is important because in the first case we obtain a point estimate for the company value, and in the second we obtain a distribution function.

We consider the optimal projection for a given model, given by the conditional expected value

$$\hat{X}_{t,h}(X_1, \ldots, X_t) = E[X_{t+h}|X_1, \ldots, X_t]. \quad [9]$$

To calculate this optimal projection the following algorithm is applied: $\hat{X}_{t,h}$ is obtained by interpreting its defining equation

$$X_{t+h} = \alpha_1 X_{t+h-1} + \alpha_2 X_{t+h-2} + \ldots + \alpha_p X_{t+h-p} + \varepsilon_{t+h} - \beta_1 \varepsilon_{t+h-1} - \ldots - \beta_q \varepsilon_{t+h-q} \quad [10]$$

in the following way:

(i) variables $X_1, \ldots, X_t$ are the actual values,
(ii) unobserved variables $X_{t+1}, \ldots, X_{t+h}$ are replaced by optimal projections $\hat{X}_{t,1}, \ldots, \hat{X}_{t,h}$,
(iii) error terms $\varepsilon_1, \ldots, \varepsilon_t$ correspond to projection errors of the optimal one step projection $X_1 - \hat{X}_{0,1}, \ldots, X_t - \hat{X}_{t-1,1}$,
(iv) error terms $\varepsilon_{t+1}, \ldots, \varepsilon_{t+h}$ are replaced by their expected value zero.

We apply this algorithm and obtain the cash flow projection for the period extending from the first quarter of 2015 to the fourth quarter of 2017.[11] In the case of Model 1, the cash flow is derived indirectly by projecting all cash flow-defining business parameters separately. In the case of Model 2, the cash flow is projected directly. Figure 6 shows the cash flow projection for both models. It can be seen that both models have a high degree of congruence. The only exception is that the cash flow peaks occurring in the fourth quarter of every year are projected more conservatively by Model 1 than Model 2.

We now study the projection based on the stochastic process itself. For that we expand the algorithm by changing the last step to

11 We consider a projection of three years because the planning system of the company comprises three years.

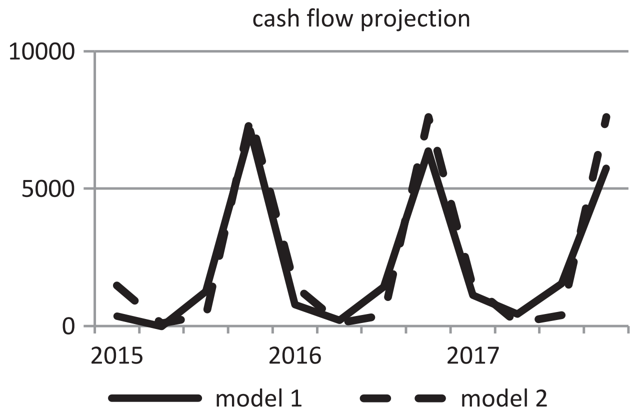

Figure 6: Cash flow projection of Model 1 and Model 2.

(iv) error terms $\varepsilon_{t+1}, \ldots, \varepsilon_{t+h}$ are normally distributed with mean zero and standard deviation σ^2.[12]

Now, eq. [10] is a stringent application of the model structure eq. [2]. The forecasted values are random variables generated by eq. [10], which follow a conditional distribution under the condition of the given data $(X_1, \ldots, X_t)$. The stochastic aspect of the point estimate approach arises from the randomness inherently contained in the data. Under the condition of the given data, there is a single point estimate. In contrast, the stochastic nature of the approach that explicitly uses stochastic process arises from the normally distributed error terms in eq. [10]. There still remains uncertainty, even under the condition of the given data. Since the forecasted values are future values of both the cash flow-defining business parameters and the cash flow itself, which are uncertain in nature, the approach based on a stochastic process is more realistic. In the next section, we derive the company value based on each approach in turn.

6 Valuation of the Company

In this section, we derive the company value for the chosen corporate plan based on Model 1 and Model 2 in turn. We distinguish between a point estimate and a distribution function for the company value.

12 Note that the standard deviation is estimated using the maximum likelihood method when fitting the model.

6.1 Valuation Based on Optimal Projection

For Model 1, we obtain a corporate plan integrating all cash flow-defining business parameters on a quarterly basis. We are able to analyse the cash flow by considering its components and control the business units responsible for any specific component. The cash flow is then derived indirectly. The complete corporate plan can be seen in Figure 7. Applying Model 2, we project the cash flow directly, so no further information except the cash flow projection is given. The cash flow plan can also be studied in Figure 7. To calculate the discount rate we use the Capital-Asset-Pricing-Model.[13] The cash flow projection given by Model 1 is slightly more conservative than the projection based on Model 2. So the present value of future cash flows and therefore the company value given by Model 1 is slightly less than the company value given by Model 2. The point estimate of company value is equal to 88 045 TEUR in the case of Model 1, and 95 478 TEUR in the case of Model 2.

Model 1 (TEUR)

	2015				2016				2017				2015	2016	2017	present value
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4				
Sales	8.292	8.205	9.246	15.473	8.754	8.452	9.406	14.691	9.124	8.678	9.542	14.032	41.216	41.303	41.376	413.551
Material	3.210	3.210	3.210	3.210	3.210	3.210	3.210	3.210	3.210	3.210	3.210	3.210	12.840	12.840	12.840	128.403
Personnel	4.730	5.017	4.779	5.164	4.769	5.031	4.786	5.119	4.795	5.035	4.798	5.085	19.690	19.705	19.713	197.104
Cash flow	352	-22	1.257	7.099	775	210	1.410	6.363	1.119	433	1.534	5.736	8.686	8.758	8.822	**88.045**

Model 2 (TEUR)

	2015				2016				2017				2015	2016	2017	present value
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4				
Cash flow	1.474	94	396	7.602	1.448	100	395	7.602	1.448	100	395	7.602	9.567	9.546	9.546	**95.478**

Figure 7: Cash flow projection and company value from Model 1 and Model 2.
Notes: We deliberately omit a consideration of taxes, as they do not change the analysis. To include taxes, we just have to multiply the cash flows and the corresponding company value by the factor (1- tax quota).
The terminal value is calculated in year 2017. In this year the company has reached a sustainable cash flow.

13 We use a risk-free interest rate, a market risk premium and the average beta factor of a peer group resulting in a discount rate of 10 %.

6.2 Valuation Based on the Stochastic Process

6.2.1 Theoretical Derivation

Thus, from the optimal projection we find a point estimate for the company value. Now we expand the analysis by considering the company value based on the stochastic process. Subject to the condition of the data, using eq. [1] and the algorithm of the last section, we can write the company value in the following form:

$$CV = \underbrace{\left(\frac{1}{1+k}, \dots, \frac{1}{k(1+k)^2}\right)}_{v} \underbrace{\begin{pmatrix} 1..1 & & 0 \\ & \ddots & \\ 0 & & 1..1 \end{pmatrix}}_{\Delta} \left[\underbrace{\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ f_{ij}(\cdot,\cdot) & & 1 \end{pmatrix}}_{A} \begin{pmatrix} \varepsilon_{t+1} \\ \vdots \\ \varepsilon_{t+T} \end{pmatrix} + \underbrace{\begin{pmatrix} f_1(\cdot,\cdot) \\ \vdots \\ f_T(\cdot,\cdot) \end{pmatrix}}_{b} \right] \qquad [11]$$

The vector v is for discounting the annual future cash flows at a discount rate of k. The matrix Δ transforms the quarterly cash flows into annual cash flows. The matrix A represents the stochastic part, and the vector b represents the deterministic part of the ARIMA model. The functions f_{ij} are polynomials of degree i–1 and the functions f_i are polynomials of degree i, which can be shown recursively by using eq. [10]. We demonstrate this process for Model 1 and Model 2 in turn:

By considering Model 1, we give an analysis of an ARIMA(3,1,1)[14] model. Applying eq. [10] for the first-order differences we get the recursion

$$X_{t+1} = (1+\alpha_1)X_t + (\alpha_2 - \alpha_1)X_{t-1} + (\alpha_3 - \alpha_2)X_{t-2} - \alpha_3 X_{t-3} + \varepsilon_{t+1} - \beta_1 \varepsilon_t. \qquad [12]$$

From this recursion we derive the functions f_{ij} and f_i which are the following polynomials:[15]

14 The reason that we choose an ARMA(3,1,1) model is that for Model 1 the only ARMA models that occur have order smaller than $p=3$, $d=1$ and $q=1$. Therefore we can derive formulas for these models from the chosen model.

15 Note that under the condition of the given data the variables $X_{\le t}$ and $\varepsilon_{\le t}$ are just numbers. Therefore they occur in the function f_i.

$$
\begin{aligned}
f_1(\alpha,\beta) &= (1+\alpha_1)X_t + (\alpha_2-\alpha_1)X_{t-1} + (\alpha_3-\alpha_2)X_{t-2} - \alpha_3 X_{t-3} - \beta_1\varepsilon_t \\
f_{21}(\alpha,\beta) &= (1+\alpha_1-\beta_1) \\
f_2(\alpha,\beta) &= \left(1+\alpha_1+\alpha_1^2+\alpha_2\right)X_t + \left(-\alpha_1-\alpha_1^2+\alpha_1\alpha_2+\alpha_3\right)X_{t-1} - \alpha_2 X_{t-2} \\
&\quad - (1+\alpha_1)X_{t-3} - (\beta_1+\alpha_1\beta_1)\varepsilon_t \\
&\vdots
\end{aligned} \qquad [13]
$$

We have to apply eq. [11] to every cash flow-defining business parameter. By considering Model 2 and applying eq. [10], the recursion is given by eq. [3]. From this recursion we get the following functions:

$$
\begin{aligned}
f_1(\phi,\theta) &= CF_{t-3} + \phi(CF_t - CF_{t-4}) - \theta\varepsilon_{t-3} \\
f_{21}(\phi,\theta) &= \phi \\
f_2(\phi,\theta) &= CF_{t-2} + \phi^2(CF_t - CF_{t-4}) - \theta\varepsilon_{t-2} - \phi\theta\varepsilon_{t-3} \\
&\vdots
\end{aligned} \qquad [14]
$$

So, by considering eq. [11], it can be seen that the company value is just a linear transformation of the normally distributed error term ε. Since a linear transformation of a normally distributed random variable is again normally distributed, we obtain the important result that the company value is normally distributed with

$$E[CV] = v\Delta b \text{ and } \quad \mathrm{Var}[CV] = \sigma^2 (v\Delta A)^T (v\Delta A) \qquad [15]$$

Table 2 shows the results of eqs [11] and [15] for all cash flow-defining business parameters and the cash flow itself in the case of Model 1, and for the cash flow in the case of Model 2.

Table 2: Theoretical and empirical expected value and variance for the present values.

	Theory using eqs [11] and [15]		Practice using simulation technique	
	Expected value	**Standard deviation**	**Expected value**	**Standard deviation**
Model 1				
Sales	413 551	78 485	413 827	78 500
Material	128 403	40 370	128 652	40 378
Personnel	197 107	11 626	197 057	11 629
Cash flow[a]	88 045	89 021	88 118	89 039
Model 2				
Cash flow	95 478	41 423	95 473	41 431

Note: [a]Note that the present value of the cash flows is the company value.

Therefore, the respective company values yielded by Model 1 and Model 2 can be analysed using tools from probability theory. It is possible to calculate the probability that the company value is larger or smaller than a certain value. In particular, it can be studied if the net effect of a strategy or an investment on the company value is larger than 0. This would mean that the strategy or investment is value-creating. In general, any instrument of value-at-risk can be applied, enabling a fundamental analysis of the company value.

6.2.2 Practical Approach Using Simulation Technique

For practical reasons, it may be more convenient to derive the company value distribution using simulation techniques instead of calculating eqs [11] and [15] analytically. For this reason, we simulate $n = 2\ 500$ cash flow projections and calculate the corresponding company value. The histogram of the company value is shown in Figure 8 for Model 1 and in Figure 9 for Model 2. The corresponding empirical results for the expected value and variance are given in Table 2.

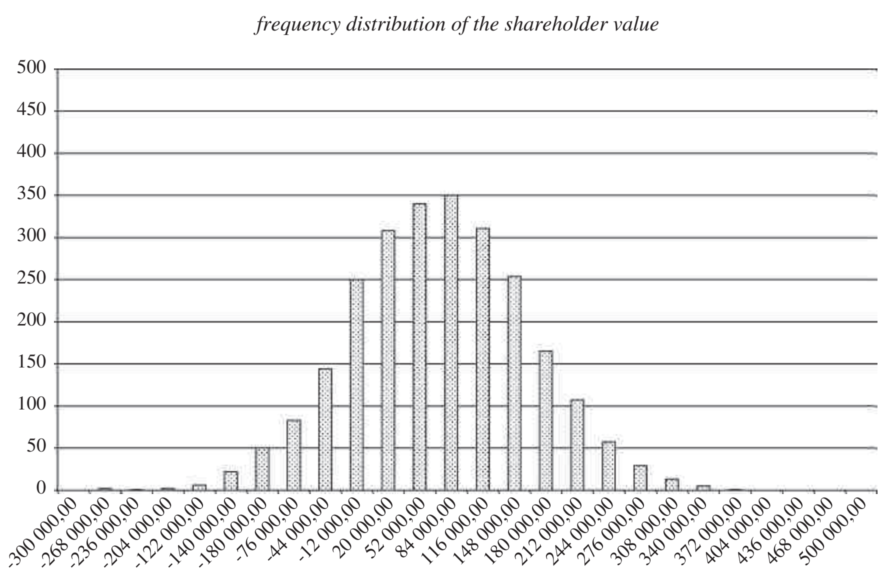

Figure 8: Histogram of company value based on Model 1.

By considering Table 2, we can state the following: due to the linearity of the expected value and $E[\varepsilon] = 0$, the company value obtained from the optimal

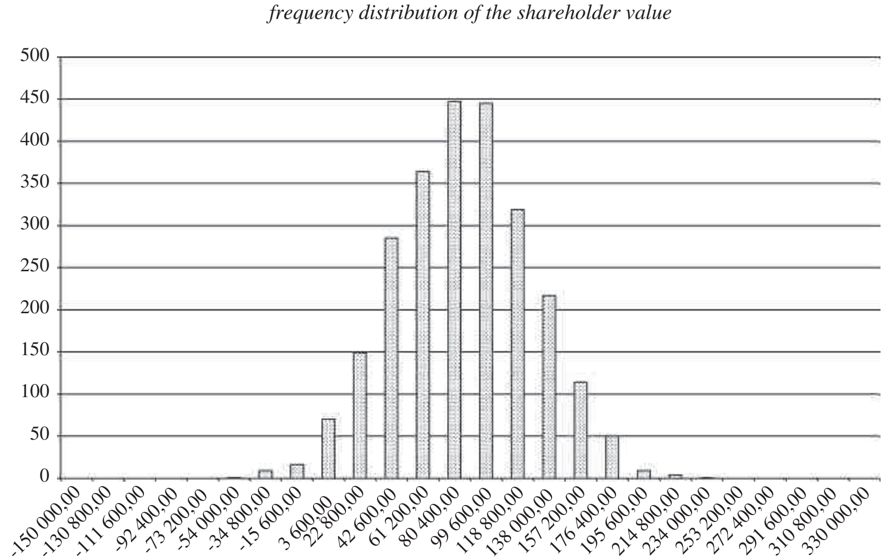

Figure 9: Histogram of company value based on Model 2.

projection and the expected value of the company value obtain from analysing the stochastic process are equal. The theoretical expected value and standard deviation calculated in eqs [11] and [15] are well-approximated by the empirical expected value and the standard deviation found by simulation. The standard deviation of the company value given by Model 1 is larger than the standard deviation given by Model 2. This is because the individual standard deviations of the cash flow-defining business parameter models add up to a relatively large standard deviation for the cash flow itself. This can also be seen when comparing Figures 8 and 9.

In summary, we have found a distribution function for every cash flow-defining parameter, i. e. sales, material-related expenditure and personnel-related expenditure, by applying Model 1. We found the theoretical distribution function using eqs [11] and [15], as well as the empirical distribution function by simulation technique. We are able to analyse every component of the cash flow using concepts in probability theory. Therefore, we can decide if the value proposition of a certain business unit is sufficient.[16] This allows for operative

16 For sales this value proposition is positive, for material-related and personnel-related expenditure it is negative. In all cases the management can define values which must not be fallen below in order to reach a certain company value.

management and controlling. Again, we see that Model 1 should be used for operative purposes in the context of value-based management. When we apply Model 2, we obtain a distribution function for the company value with a relatively small standard deviation. This can be seen in Figure 9 as well as in Table 2. The accuracy of this estimate of company value is quite high. When estimating financial ratios in relation to company value, or the company value itself, the results provided by this model are of high quality. Again, we see that Model 2 should be used for strategic purposes in the context of value-based management.

7 Extrapolation Problem

In this section we discuss from a theoretical and practical perspective the situations, in which pure extrapolation technique is a suitable way of projecting future cash flows and how we can handle a structural break with the consequence, that we have to find other than pure extrapolation techniques.

7.1 Pure Extrapolation Technique

We shall now analyse where a pure extrapolation technique is a suitable way of projecting future cash flows with a high forecasting accuracy. Autoregressive moving average and multivariate regression models are considered separately and in combination to find the optimal forecasting method.

7.1.1 Autoregressive Moving Average Model

Lorek and Willinger (2011) provide an analysis concerning the forecasting accuracy of the Brown-Rozeff model, which corresponds to our Model 2 by applying pure extrapolation technique to derive cash flow projection. They calculate 1-through 20-step-ahead quarterly cash flow predictions for a large sample of companies and achieve a good forecasting accuracy on the basis of pure extrapolation technique.[17] This high forecasting accuracy is valid for short term as well as long term predictions. Furthermore, the model of Brown-Rozeff is

17 MAPEs of the Brown-Rozeff model for 1- through 20-step-ahead prediction are between 58 % and 69 %, see Lorek and Willinger (2011, 80).

compared to a multivariate regression model in terms of forecasting accuracy.[18] Using Wilcoxon rank-sum test the forecasting accuracy of the Brown-Rozeff model based on extrapolation technique is significantly better than the forecasting accuracy of a multivariate regression model.[19] Thus in general, cash flow projection based on the Brown-Rozeff model and applying extrapolation technique is verified as being a suitable way of deriving future cash flows in the short term as well as the long term perspective for a large sample of companies.

Lorek and Willinger are interested in analysing the ability of their models to predict quarterly cash flows. Therefore they benchmark their models by calculating MAPEs on a quarterly basis. We are interested in evaluating a company based on yearly future cash flows. The quarterly distribution of these yearly cash flows is irrelevant for the company value. Therefore we benchmark our models using MAPEs on a yearly basis. Beside yearly MAPEs we also provide quarterly MAPEs to show, that all results are valid on the basis of yearly and quarterly MAPEs. Since quarterly excesses and deficits may partly balance each other, yearly MAPEs are generally smaller than quarterly MAPEs.[20]

When considering MAPEs of Model 1 and Model 2 on a quarterly basis we have a MAPE of 83 % for the cash flow in Model 1, which is comparable to the pooled MAPE of the multivariate regression model analysed by Lorek and Willinger and a MAPE of 61 % for the cash flow in Model 2, which is comparable to the pooled MAPE of the Brown-Rozeff model studied by Lorek and Willinger.[21] Therefore our results and the results provided by Lorek and Willinger are consistent.

When evaluating a company in a situation with no structural break, we may anticipate the following: if we are interested in the value of the company and therefore focus on the yearly future cash flows, we should archive a forecasting accuracy in the range of the yearly MAPEs as reported in this paper. If we are interested in the exact future business projection and therefore focus on quarterly future cash flows we should obtain a forecasting accuracy in the range of

18 MAPEs of the multivariate regression model for 1- through 20-step-ahead prediction are between 75 % and 83 %, see Lorek and Willinger (2011, 80).

19 The significance level of Wilcoxon rank-sum test is 1 %.

20 When calculating the yearly MAPE and therefore comparing yearly actual and fitted values, we may consider the percentage difference between a yearly actual and a fitted value as a weighted sum of quarterly percentage differences between actual and fitted values with weights equal to quarterly actual values. Our models fit very well to the data especially in the main quarter, which is the fourth quarter. Therefore quarterly percentage differences between actual and fitted values are small for observations with a high weight. Thus the yearly MAPE tends to be small because the models are suitable for deriving the company value.

21 See Lorek and Willinger (2011, 80).

the quarterly MAPEs as reported in this paper or, equivalent, as provided by Lorek and Willinger.

7.1.2 Multivariate Regression Model

Since the multivariate regression model considered by Lorek and Willinger has a poor forecasting accuracy when applied to our software company, we develop a new multivariate regression model. This model consists of a linear trend variable and four seasonal variables representing the seasonal business. This multivariate regression model has a suitable forecasting accuracy.[22]

Lorek and Willinger do not include autoregressive techniques applied to the error terms in their multivariate regression model. For that reason we are interested in the question if it is possible to improve the forecasting accuracy of the multivariate regression model when applying autoregressive techniques to the error terms. Therefore we study a combination of an autoregressive moving average and a multivariate regression model.

7.1.3 Combination of Autoregressive Moving Average and Multivariate Regression Model

We now apply autoregressive techniques to the error terms of our multivariate regression model and compare the forecasting accuracy of the multivariate regression model with and without autoregressive techniques. The results are provided in Table 3. We realize that the forecasting accuracy of the multivariate

Table 3: Yearly MAPEs of Model 1, Model 2 and multivariate regression model with and without autoregressive technique applied to error terms.

	Model 1 and Model 2		Multivariate regression model	
	Model 1	Model 2	Autoregressive techniques applied to error terms	Independent identically distributed error terms
Sales	1 %		9 %	10 %
Material	4 %		16 %	16 %
Personnel	0 %		2 %	3 %
Cash flow	5 %	1 %	15 %	17 %

Note: The table for the quarterly MAPEs shows a comparable relation between the different models.

22 MAPEs of this model are provided in Table 3.

regression model with autoregressive techniques applied to the error terms is slightly better than the forecasting accuracy of the multivariate regression model without autoregressive techniques. This result is valid for all cash flow-defining business parameters and the cash flow itself. Therefore we may expect that when applying autoregressive techniques to the error terms the multivariate regression model can be improved in terms of forecasting accuracy. However, the forecasting accuracy of our Model 1 and Model 2 is still superior to both multivariate regression models, as can be seen in Table 3.

7.2 Structural Break

Besides pure extrapolation technique a structural break could also occur in the business requiring an adaptation of the model to the actual business situation. We differentiate between a structural break in the business model and a structural break in the economic scenario. Furthermore we discuss how to detect and to handle a structural change in the autoregressive moving average model.

7.2.1 Structural Break in the Business Model

A break in the business model occurs for example if the company invests in a new product or expands its business to a new market. For this new strategy a business plan must exist to decide whether the new strategy with its corresponding investments is value-creating or not.[23] Therefore it is possible to calculate exactly how much the cash flow-defining business parameters such as sales, material-related and personnel-related expenditure are influenced. If a business parameter is influenced by x% we apply the following algorithm for adapting the model to the new business situation: shift Y_t to $\tilde{Y}_t = (1+x)Y_t$ for all past t. We then apply the original model with the original parameter estimation to the shifted data. This algorithm results in a projection of the business parameter, which is adapted to the new business situation, i. e. the original forecast of the business parameter derived on basis of pure extrapolation is increased or decreased by x%.[24] Now we are able to calculate the company value of the new business model.

23 A strategy with its corresponding investments is realized if and only if the present value of the future cash flows is positive.

24 This algorithm is valid for a non-differentiated autoregressive moving average process as well as for a differentiated autoregressive moving average process.

We demonstrate the theoretically described method with reference to our software company. The company is currently in negotiations with another software company concerning a strategic partnership. This strategic partnership would cause a structural break in the business model. We now calculate the new strategic company value based on this strategic partnership. The company has provided a business plan to decide if this partnership is value-creating or not in order to convince its shareholders. The result of this business model is that due to cross-selling possibilities sales increase by 2 %, due to cost synergies material-related expenditure decreases by 2 % and personnel-related expenditure decreases by 1 %. Applying the algorithm to the data we get the cash flow projection as shown in Figure 10. It can be seen that the additional cash flows using Model 1 are uniformly added to all quarterly cash flows, while additional cash flows using Model 2 are summated disproportionately highly in the fourth quarter due to high peaks in the fourth quarter.

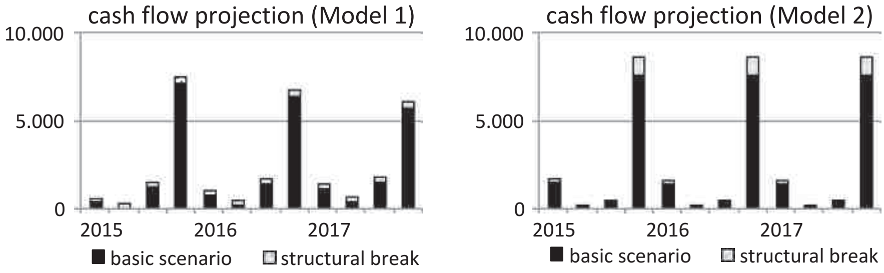

Figure 10: Cash flow projection of Model 1 and Model 2.

When calculating the new strategic company value we add the original standalone value we already know and the present value of the additional cash flows resulting from the new strategic partnership. For Model 1 we have 88 045 TEUR + 12 810 TEUR = 100 855 TEUR and for Model 2 we obtain 95 478 TEUR + 13 367 TEUR = 108 845 TEUR in the deterministic case. These values are also the expected values for the stochastic case. The standard deviations are 90 036 for Model 1 and 47 222 for Model 2.

7.2.2 Structural Break in the Economic Scenario

A break in the economic scenario occurs for example if the economic demand significantly changes. This situation can be analysed using change point analysis.[25] Change point analysis detects a significant change in an economic parameter by comparing its development in the past. Let $e_1,\ldots, e_t$ be a sequence of independent[26] normal random variables with expected value and variance $(\mu_1, \sigma^2), \ldots, (\mu_t, \sigma^2)$ representing the development of an economic parameter. We are interested if there is a change point in the mean of this economic parameter. Therefore we test the hypothesis of stability, the null hypothesis,

$$H_0 : \mu_1 = \ldots = \mu_t = \mu$$

versus the alternative

$$H_1 : \mu_1 = \ldots = \mu_k \neq \mu_{k+1} = \ldots = \mu_t,$$

where k is the unknown location of a single change point. We define

$$\bar{e}_k = \frac{1}{k}\sum_{i=1}^{k} e_i, \bar{e}_{t-k} = \frac{1}{t-k}\sum_{i=k+1}^{t} e_i, \bar{e} = \frac{1}{t}\sum_{i=1}^{t} e_i \text{ and}$$

$$S_k = \sum_{i=1}^{k} (e_i - \bar{e}_k)^2 + \sum_{i=k+1}^{t} (e_i - \bar{e}_{t-k})^2, S = \sum_{i=1}^{t} (e_i - \bar{e})^2, V_k = S - S_k. \qquad [16]$$

The test statistic U is derived on basis of the likelihood procedure and is defined as

$$U = \max_{1 \le k \le t-1} \sqrt{V_k}. \qquad [17]$$

It is possible to calculate the exact and asymptotic H_0 probability density function of U.[27] Therefore we are able to construct a test of a structural break in the economic parameter: H_0 is rejected if the test statistic U is larger than the $(1-\alpha)$ quantile of the H_0 probability distribution.

If a change point occurs it is necessary to estimate the impact on all business parameters. To do this an iterative process with the management of the company and the valuation experts should be applied: the adopted model using the mathematical algorithm described below should be executed by the valuation experts resulting in a new company value. This result should be discussed with the management to assess whether it is plausible. During this

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25 An introduction to change point analysis is provided in Chen and Gupta (2012). Change point analysis for autoregressive moving average processes is described elaborately in Muhsal (2013).

26 If the random variables are not independent, the change point analysis can be generalized to model this situation.

27 The exact H_0 probability density function is shown in Chen and Gupta (2012, 10–14) and the asymptotic H_0 probability density function is derived in Chen and Gupta (2012, 14–19).

step the experience of the management should be used to quantify the impact of an economic parameter on a business parameter. If the result is not plausible, the analysis should be used to redefine this impact on the business parameters, i. e. to improve the parameter setting in the mathematical algorithm. This cycle should be repeated until the management and the valuation experts are satisfied by the model results.

The mathematical algorithm to suitably model a structural break in the economic scenario is as follows: the interpretation of a normalized economic parameter e is that we expect an increasing business development for $e > 100$, a stable business for $e = 100$ and a decreasing business development for $e < 100$. We first fit the model before the structural break in the economic parameter by deriving the trend and the seasonal part in the following way: let $X_1, \ldots, X_t$ be the actual values of the business parameter. We solve

$$\sum_{i=1}^{t} [X_i - (a_{\text{old}} i + b_{\text{old}})]^2 \to \min_{(a_{\text{old}}, b_{\text{old}})} \qquad [18]$$

to obtain the trend function trend(i) = $a_{\text{old}}\ i + b_{\text{old}}$. For the remaining seasonal part X_i − trend(i) we fit an ARMA(p, q) model. When projecting the business parameter after the structural break in the economic parameter we have to adopt the trend part due to this structural break. For this we need a linear function, which maps $e_{\text{old}} \mapsto a_{\text{old}}$, $100 \mapsto 0$ and $e_{\text{new}} \mapsto a_{\text{new}}$. The following map f complies with these requirements:

$$f(e) = \frac{e - 100}{e_{\text{old}} - 100} a_{\text{old}} \qquad [19].$$

Now we are able to project the business parameter after the structural break in the economic parameter by using the new trend function

$$X_{t+h} = \text{trend}(h) + \text{season}(h), h = 1, \ldots, T - t, \qquad [20]$$

where $trend(h) = a_{\text{new}} h + \underbrace{(a_{\text{old}} t + b_{\text{old}})}_{= b_{\text{new}}}$, $h = 1, \ldots, T - t$ and season(h): projection of the ARMA(p, q) model.

Now we apply this theory to practical data. For this we consider the two economic parameters purchase manager index and business climate index. These two economic parameters are highly relevant in order to estimate the economic demand and therefore the development of future company sales. If there is a structural break in these economic parameters we can expect an impact on the business parameter sales. The development of both economic parameters is shown in Figure 11.

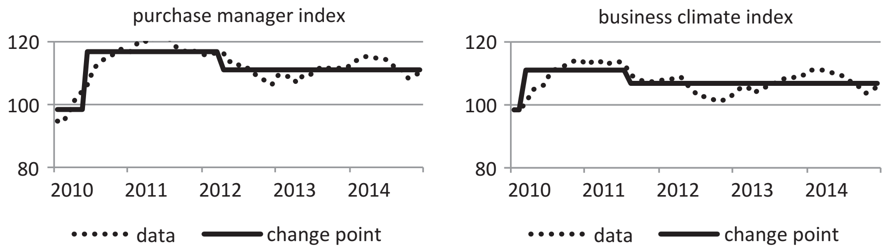

Figure 11: Development of the economic parameters purchase manager index and business climate index.

Applying the described change point analysis with a significance level of 5 % we find change points in 2010 second quarter and 2012 second quarter for the purchase manager index and 2010 first quarter and 2011 third quarter for the business climate index. This result is plausible, because there is first a change in business climate and then as a consequence purchase managers decide to adopt their purchasing policy.

We conclude that when evaluating a company in the years 2013 or 2014 there is no need to adopt the model due to a structural break in an economic parameter. A pure extrapolation technique appears to be suitable. This is the reason why the forecasting accuracy of Model 1 and Model 2 is very high as we have seen in section 5.3.2. Also the cash flow projection on basis of pure extrapolation technique in section 5.4 is appropriate. However, if we would evaluate a company in the year 2012 we had to apply the described mathematical algorithm adapting the trend variable a_{old} to a_{new}. For this we use eq. [19] with $e_{\text{old}} = 117{,}0$ and $e_{\text{new}} = 111{,}2$ for the purchase manager index and derive $a_{\text{new}} = 0{,}66\ a_{\text{old}}$.

7.2.3 Structural Change in the Autoregressive Moving Average Model

We now discuss how to detect and to handle a structural change in the autoregressive moving average model.[28] We had a structural break in the business model or in the economic scenario at time t, which is the actual time for evaluating the company in Sections 7.2.1 and 7.2.2, respectively. Since these structural breaks have an impact on the projection of a certain business parameter, we had to quantify this impact

28 Clements (2006, 605–657) describes methods for detecting a structural change in an autoregressive moving average model and forecasting when there are structural breaks.

without having information contained in the past data of the business parameter. Now the situation is different: we fit an autoregressive moving average model to the past data of a certain business parameter and test if there is a structural change in the parameters of this model. Thus, information about the structural change is contained in the data.

We consider the Chow test,[29] which tests if there is a structural change or non-constancy in the parameters of the autoregressive moving average model. The test compares our fitted model eq. [2] to two sub models

$$X_t = \alpha_1^1 X_{t-1} + \alpha_2^1 X_{t-2} + \ldots + \alpha_p^1 X_{t-p} + \varepsilon_t - \beta_1^1 \varepsilon_{t-1} - \ldots - \beta_q^1 \varepsilon_{t-q}, t = 1 \ldots t_0, \qquad [21a]$$

$$X_t = \alpha_1^2 X_{t-1} + \alpha_2^2 X_{t-2} + \ldots + \alpha_p^2 X_{t-p} + \varepsilon_t - \beta_1^2 \varepsilon_{t-1} - \ldots - \beta_q^2 \varepsilon_{t-q}, t = t_0 + 1 \ldots T \qquad [21b]$$

and tests the hypothesis

$$H_0 : \left(\alpha_1^1, \ldots, \alpha_p^1, \beta_1^1, \ldots, \beta_q^1\right) = \left(\alpha_1^2, \ldots, \alpha_p^2, \beta_1^2, \ldots, \beta_q^2\right)$$

versus the alternative

$$H_1 : \left(\alpha_1^1, \ldots, \alpha_p^1, \beta_1^1, \ldots, \beta_q^1\right) \neq \left(\alpha_1^2, \ldots, \alpha_p^2, \beta_1^2, \ldots, \beta_q^2\right)$$

Defining S_a as the sum of squared residuals of eq. [21a], S_b as the sum of squared residuals of eq. [21b] and S as the sum of squared residuals of eq. [2], the H_0 probability distribution function of the test statistic V

$$V = \frac{S - (S_a + S_b)}{(S_a + S_b)/(t_0 - 2(p + q))} \qquad [22]$$

is an F-distribution. Therefore we are able to construct a test of a structural change or non-constancy in the parameters: H_0 is rejected if the test statistic V is larger than the (1–α) quantile of the H_0 probability distribution.

When applying the Chow test with a significance level of 5 % to our autoregressive moving average models for the software company, H_0 is not rejected for all suitable choices of t_0. This means that there is no structural change or non-constancy in the parameters. The results of this section and section 7.2.2 are consistent in the following way: on basis of change point analysis applied to economic parameters we expect ex ante a reduced growth rate of demand in the year 2012. Ex post we actually have a reduced growth rate of the company sales. But this reduction is moderate, because the company was able to improve its price-performance ratio in its peer group and therefore partially balance this changed

29 The Chow test is provided in Clements (2006, 622) and Steffensmeier et al. 2014, 65–66).

economic situation. As a consequence the reduction of the growth rate is not significant, therefore we do not observe any structural change in the model.

In the case of a structural change in the model we have two possibilities when forecasting business parameters: on the one hand we use the model [21b] after the structural break at time t_0 for projecting the business parameter. On the other hand we model further structural breaks. In a deterministic approach the times of a structural break and the change in the model parameters are determined. A suitable calculation for the time is $2t_0$, $3t_0$,..., T and for the change in the parameters is $\Delta = \left(\alpha_1^2 - \alpha_1^1, \ldots, \alpha_p^2 - \alpha_p^1,\ \beta_1^2 - \beta_1^1, \ldots, \beta_q^2 - \beta_q^1\right)$. In a stochastic approach times and parameter changes are random. A possible calculation is to assume that the random variables representing time and parameter changes are normally distributed and to identify the expected values with the values of the deterministic approach.

8 Expansion of the Model

8.1 Data

We now consider a machine building company in order to expand the model to other industries. This company produces rolls for the printing and packaging industry. The business model is based on a classical project-based business. Again, the data we will analyse is from the period 2010 to 2014. Sales result from different projects of the company with the printing and packaging industry. Material-related expenditure occurs when the company purchases components of the rolls. Personnel-related expenditure refers to the salaries of the workers producing the rolls. These workers are hired separately for every project. Therefore material and personnel-related expenditures are variable costs. Furthermore there are expenditures for maintenance of the production plants. These expenditures are fixed costs. Other income and expenditure are negligibly small, so that the cash flow mainly results from these business parameters. Additionally, we include taxes in the model. We apply our model to this manufacturer by fitting Model 1 to the described business parameters as key performance indicators to project the future cash flow indirectly and Model 2 to the cash flow to project the future cash flow directly. We provide the projection of the cash flows and the valuation of the company. In particular, we review whether the thesis that Model 1 should be used for operative purposes and Model 2 for strategic purposes of value-based management still remains valid for the machine building company.

8.2 Projection of Cash Flows and Valuation of the Company

When fitting Model 1 and Model 2 to the data the residual analysis shows that the residuals are normally distributed, homoscedastic and independent.[30] When considering Figure 12, we see that Model 1 is a good fit for the cash flow-defining business parameters and for the derived cash flow itself. Also Model 2 provides a good fit for the data. The corresponding MAPEs confirm this graphical analysis.[31]

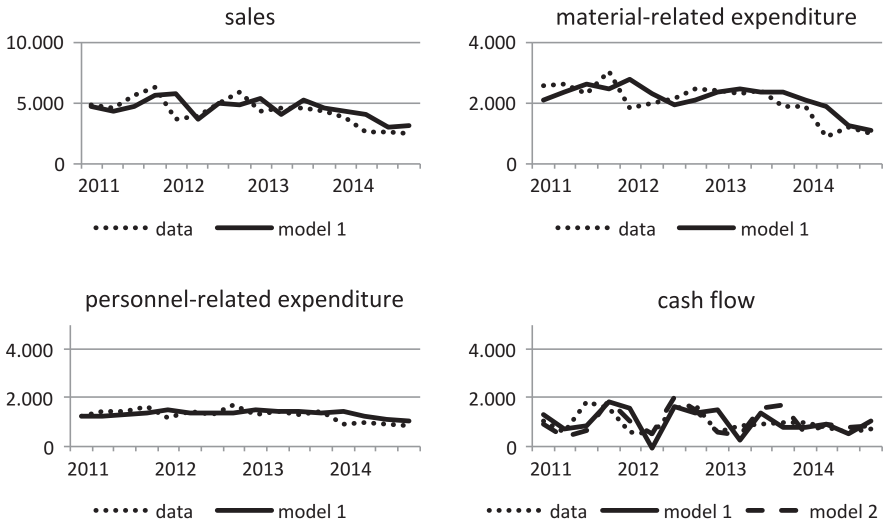

Figure 12: Model fitting of Model 1 and Model 2.

In terms of forecasting accuracy Model 2 performs slightly better than Model 1.[32] The forecasting accuracy of Model 2 is nearly perfect. Results for the cash flow projection and company valuation are shown in Figure 13 for the deterministic

30 The analysis was made on basis of residual plots and corresponding statistical tests.
31 We have yearly MAPEs between 10 % and 13 % for Model 1 and a yearly MAPE of 18 % for Model 2. The quarterly MAPEs are between 15 % and 43 % for Model 1 and 36 % for Model 2.
32 We have a yearly MAPE of 0 % for the cash flow in Model 2 and a slightly higher yearly MAPE of 15 % for the cash flow in Model 1. The quarterly MAPE for the cash flow in Model 2 is 41 % and for the cash flow in Model 1 is 43 %. These forecasting accuracies are slightly better than the forecasting accuracies of the multivariate regression model and the Brown-Rozeff model analysed by Lorek and Willinger.

Model 1 (TEUR)

	2015				2016				2017				2015	2016	2017	present value
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4				
Sales	2.550	2.603	2.599	2.578	2.582	2.590	2.588	2.585	2.586	2.587	2.586	2.586	10.330	10.344	10.345	103.432
Material	1.085	1.049	1.064	1.058	1.060	1.059	1.060	1.059	1.059	1.059	1.059	1.059	4.256	4.238	4.238	42.395
Personnel	957	957	957	957	957	957	957	957	957	957	957	957	3.826	3.826	3.826	38.262
Maintenance	100	100	100	100	100	100	100	100	100	100	100	100	400	400	400	4.000
Cash flow before tax	408	498	479	464	465	474	471	469	470	471	470	470	1.848	1.879	1.881	18.774
Tax	118	144	139	134	135	137	137	136	136	137	136	136	536	545	545	5.445
Cash flow after tax	289	353	340	329	330	337	335	333	333	334	334	334	1.312	1.334	1.335	**13.330**

Model 2 (TEUR)

	2015				2016				2017				2015	2016	2017	present value
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4				
Cash flow before tax	966	667	428	586	964	666	428	586	964	666	428	586	2.647	2.645	2.645	26.454
Tax	280	193	124	170	280	193	124	170	280	193	124	170	768	767	767	7.672
Cash flow after tax	686	473	304	416	685	473	304	416	685	473	304	416	1.880	1.878	1.878	**18.782**

Figure 13: Cash flow projection and company value from Model 1 and Model 2.
Note: The terminal value is calculated in year 2017. In this year the company has reached a sustainable cash flow.

Table 4: Theoretical and empirical expected value and variance for the present values.

	Theory using eqs [11] and [15]		Practice using simulation technique	
	Expected value	Standard deviation	Expected value	Standard deviation
Model 1				
Sales	103 432	68 421	103 844	68 435
Material	42 395	36 895	42 215	36 902
Personnel	38 262	23 013	38 923	23 018
Maintenance	4 000	0	4 000	0
Cash flow before tax	18 774	81 070	18 706	81 086
Tax	5 445	6 818	5 425	6 819
Cash flow after tax	13 330	40 867	13 281	40 875
Model 2				
Cash flow before tax	26 454	71 159	26 445	71 173
Tax	7 672	5 984	7 669	5 986
Cash flow after tax	18 782	35 871	18 776	35 878

case and in Table 4 for the stochastic case. Due to a more conservative projection of the cash flows in the peak quarters the company value given by Model 1 is again smaller than the company value given by Model 2. Furthermore, it can be seen that the standard deviation of the company value given by Model 1 is again larger than the standard deviation given by Model 2.

Thus, the thesis we have stated for the software company that Model 1 should be used for operative purposes and Model 2 for strategic purposes in the context of value-based management is also confirmed for the machine building company.

8.3 Shock Scenario

Overall, the forecasting accuracy is an important measure to evaluate a cash flow projecting model in the context of company valuation. The reason for this is, that the company value is the sum of discounted future cash flows and these future cash flows have to be estimated on basis of the past cash flow development. But there exist situations, in which although the forecasting accuracy of a cash flow model is highly verified for past data the model is entirely unsuitable to project future cash flows. This is the case if the planning premises have changed fundamentally. Therefore causal factors, which impact the cash flow projection, are not included in the model. We define this scenario as a shock scenario, if the planning premises have changed quickly, significantly and sustainably. A shock scenario occurs for example if substitute goods have been developed or many new competitors have entered the market. In these scenarios planning premises for company sales have changed quickly, significantly and sustainably.[33] In a shock scenario the old planning premises have been lost and have been replaced by new premises. Thus, the overall planning basis of the model has changed in order to include all factors in the model, that we know causally impact company performance. Therefore planning with new premises includes all critical causal variables any manager needs to consider in maintaining company value.[34]

We now demonstrate this shock scenario for our machine building company. The company possesses a subsidiary company, which produces rolls for

33 Also a terror attack is normally a shock scenario: Due to reduced consumer confidence demand changes quickly, significantly and sustainably. Therefore company sales projection has to be made on basis of these new premises.

34 This economic situation is comparable to the following astronomical situation: the Ptolemaic model based on epicycles very precisely predicts the motion of the planets and constellations without reference to the causal impact of gravity. The model was based on the false premise that the earth was the center of the universe. Therefore causal factors which impact the motion of the planets and constellations are not included in the model. The predictive accuracy of the model was one of the reasons that it held sway for over a thousand years. But later models (specifically Newton's law of universal gravitation which included the causal force of gravity) provided not only accurate predictions but an accurate picture of a non-geocentric universe. Therefore planning with new premises includes all critical causal variables we have to consider when analyzing the astronomical structure of the universe.

the oil industry. These rolls are used for drilling machines in oil production. The subsidiary company expects high growth rates, because there is a high demand for oil in the future and new oil pools have to be made accessible. But due to the fracking technology, a substitute technology, this demand can be satisfied alternatively. The rolls of the company are not used for the fracking technology. The planning premises have changed quickly, significantly and sustainably. Therefore we have a shock scenario. The model, which is used for the cash flow projection has a high forecasting accuracy analysed for past data, but it is absolutely unsuitable to project further cash flows. The reason is that it does not include the main factor that we know causally impact company performance. Therefore the management of the company and the management of peer group companies have been asked which consequences they anticipate due to these new planning premises. They anticipate a reduction of 50 % for sales, 50 % for material-related expenditure because these are variable costs and 40 % for personnel-related expenditure because these are predominantly variable costs. After this one-off effect moderate growth rates are expected on a reduced basis. This shock scenario model includes all critical variables that any manager needs to consider in maintaining company value. The sales projection can be seen in Figure 14. The company value in the shock scenario amounts to 7 258 TEUR, which is consistently smaller than the original company value of 19 773 TEUR.

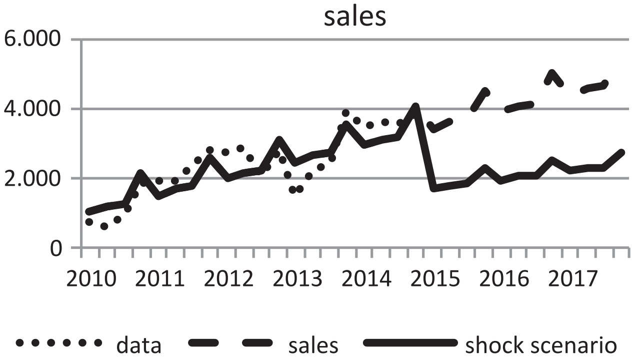

Figure 14: Shock scenario.

9 Conclusion

We summarise the results of this paper from both a theoretical and a practical perspective. From the scientific perspective, our main result expands on both a new, indirect approach, and a well-known, direct approach to stochastically predicting cash flows by means of the corresponding distribution function for the company value. We have shown a correlation between the stochastic cash flows and the corresponding distribution function for the company value. This model is essentially applicable to every company of any industry sector using empirical data. We have shown that the resulting distribution functions for the company value are normal distribution functions with known expected value and variance. Therefore, the company value can be analysed in detail using tools in probability theory. The above-stated objective of this paper has been achieved.

From the practical perspective, all instruments of value-at-risk analysis are applicable to normal distribution functions derived in the above discussion. It is possible to calculate the probability that the company value is larger or smaller than a certain value. The management need only to define a lower limit that prospective strategies or investments must exceed. Then, the probability that the strategy or investment in question outperforms this value objective may be calculated. In particular it can be determined whether a certain strategy or investment is value-creating by setting the lower limit to zero. On this basis, a rational evaluation of different strategies or investments and therefore successful value-based management is possible.

10 Further Research

On the basis of the analysis provided in this paper, it would be interesting to study empirically whether model structure and parameter estimation remain stable when considering companies over a whole industry sector. In this case, an industry model could be derived. By applying the results of this paper, the company value for such a typical representative of a given industry could be calculated. This value could be used to apply benchmarking analysis to a single company, as compared to this representative.

A further interesting area of application is value-at-risk analysis. The normal distributions that the company value is generally assumed to follow in this field of study acquire a theoretical basis from the results shown in this paper. The company value can be analysed more deeply, and practical questions regarding the business model such as the value of a market, the value of a customer or the value of a product might be answered more precisely by applying a wide range of value-at-risk instruments.

References

Bagheri, A., P. Mohammadi, and M. Akbari. 2014. "Financial Forecasting Using ANFIS Networks with Quantum-Behaved Particle Swarm Optimization." *Expert Systems with Applications* 41 (14):6235–50.

Brown, L., and M. Rozeff. 1979. "Univariate Time-Series Models of Quarterly Accounting Earnings per Share: A Proposed Model." *Journal of Accounting Research* 17 (1):179–89.

Chen, S. 2007. "Measuring Business Cycle Turning Points in Japan with the Markov Switching Panel Model." *Mathematics and Computers in Simulation* 76 (4):263–70.

Chen, J., and A. Gupta. 2012. *Parametric Statistical Change Point Analysis*. New York: Springer.

Clements, M. 2006. "Forecasting with Breaks." In *Handbook of Economic Forecasting*, edited by G. Elliott, C. Granger, and A. Timmermann, Volume 1, 605–57. North Holland.

Faroqi, A. 2014. "ARIMA Model Building and Forecasting on Imports and Exports of Pakistan." *Pakistan Journal of Statistics and Operation Research* 10 (2):157–68.

Golyandina, N., and A. Korobeynikov. 2014. "Basic Singular Spectrum Analysis and Forecasting with R." *Computational Statistics and Data Analysis* 71 (3):934–54.

Hatting, M., and D. Uys. 2014. "In-Season Retail Sales Forecasting Using Survival Models." *Orion* 30 (2):59–71.

Hülss, J., N. Vogel, P. Pohl, D. Ratz, and R. Küstermann. 2012. "Gaussian Distributed Shareholder Value as a Tool for Value Based Management – Business Horizon." *Journal of Business and Policy Research* 7 (3):123–39.

Hwang, S. 2010. "Cross-Validation of Short-Term Productivity Forecasting Methodologies." *Journal of Construction Engineering and Management* 136 (9):1037–46.

Kim, M., and W. Kross. 2005. "The Ability of Earnings to Predict Future Operating Cash Flows Has Been Increasing – Not Decreasing." *Journal of Accounting Research* 43 (5):753–80.

Lorek, K., and G. Willinger. 2008. "Time-Series Properties and Predictive Ability of Quarterly Cah Flows." *Advances in Accounting* 24 (1):65–71.

Lorek, K., and G. Willinger. 2009. "New Evidence Pertaining to the Prediction of Operating Cash Flows." *Review of Quantitative Finance and Accounting* 32 (1):1–15.

Lorek, K., and G. Willinger. 2011. "Multi-Step-Ahead Quarterly Cash-Flow Prediction Models." *Accounting Horizons* 25 (1):71–86.

McIntosh, W. 1990. "Forecasting Cash Flows: Evidence From the Financial Literature." *The Appraisal Journal* 78 (1):221–9.

Muhsal, B. 2013. "Change-Point Methods for Multivariate Autoregressive Models and Multiple Structural Breaks in the Mean." Karlsruhe.

Steffensmeier, J., J. Freeman, M. Hitt, and J. Pevehouse. 2014. *Time Series Analysis for Social Sciences*. New York: Cambridge University Press.

Vogt, A., E. Mattfeldt, G. Satzger, L. Lüders, M. Piper, O. Gehb, and W. Jones. 2014. "Analytical Support for Predicting Cost in Complex Service Delivery Environments." *IBM Journal of Research and Development* 58 (4):1–10.

Yip, H., H. Fan, and Y. Chiang. 2014. "Predicting the Maintenance Cost of Construction Equipment: Comparison Between General Regression Neuronal Network and Box-Jenkins Time Series Models." *Automation in Construction* 38 (2):30–8.

Zadeh, N., M. Sepehri, and H. Farvaresh. 2014. "Intelligent Sales Prediction for Pharmaceutical Distribution Companies: A Data Mining Based Approach." *Mathematical Problems in Engineering* 31 (1):1–15.